DGL-System mit EW, EU, VdK

A. 14.4

$$y_1' = 3y_1 + 3y_2 + e^{4t}$$
$$y_2' = 3y_1 - 5y_2 + e^{-6t}$$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} e^{4t} \\ e^{-6t} \end{pmatrix}$$

EW $\Rightarrow \begin{vmatrix} 3-\lambda & 3 \\ 3 & -5-\lambda \end{vmatrix} = 0 = (3-\lambda)(-5-\lambda) - 9$

$\rightsquigarrow \lambda^2 + 2\lambda - 24 = 0$

$\lambda_{1,2} = -1 \pm 5, \ \lambda_1 = 4, \lambda_2 = -6$

EV: $\lambda_1 = 4$:

$$\begin{array}{cc|c} -1 & 3 & 0 \\ 3 & -9 & 0 \end{array}$$

$\rightsquigarrow x_1 = 3\tilde{t}_1$
$\rightsquigarrow x_2 = \tilde{t}_1$ $\Rightarrow \vec{X}_1 = (3,1)\cdot\tilde{t}_1$

$\lambda_2 = -6$:

$$\begin{array}{cc|c} 9 & 3 & 0 \\ 3 & 1 & 0 \end{array}$$

$\rightsquigarrow x_1 = -1/3\,\tilde{t}_2$
$\rightsquigarrow x_2 = \tilde{t}_2$ $\Rightarrow \vec{X}_2 = (-1,3)\,\tilde{t}$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C_1\begin{pmatrix} 3 \\ 1 \end{pmatrix}e^{4t} + C_2\begin{pmatrix} -1 \\ 3 \end{pmatrix}e^{-6t}$$

$$y_H = \begin{pmatrix} 3e^{4t} & -e^{-6t} \\ e^{4t} & 3e^{-6t} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

VdK:

$$\begin{array}{cc|cl} \dot{C}_1 & \dot{C}_2 & & \\ \hline 3e^{4t} & -e^{-6t} & e^{4t} & \\ e^{4t} & 3e^{-6t} & e^{-6t} &]_{-3} \\ \hline & -10e^{-6t} & e^{4t} - 3e^{-6t} & \end{array}$$

$$\dot{C}_2 = \frac{e^{4t} - 3e^{-6t}}{-10e^{-6t}} = \frac{3}{10} - \frac{1}{10}e^{10t}$$

$$C_2 = \frac{3}{10}t - \frac{1}{10}\cdot\frac{e^{10t}}{10}$$

$$\dot{C}_1 = \frac{e^{-6t} - 3e^{-6t}\left(\frac{3}{10} - \frac{1}{10}e^{10t}\right)}{e^{4t}} = \frac{1}{10}e^{-10t} + \frac{3}{10}$$

$$C_1 = \frac{3}{10}t + \frac{1}{10}\frac{e^{-10t}}{-10}$$

$$y_{1S} = \left(\frac{3}{10}t - \frac{e^{-10t}}{100}\right)3e^{4t} + \left(\frac{3}{10}t - \frac{e^{10t}}{100}\right)\left(-e^{-6t}\right)$$

$$= \frac{1}{100}e^{4t} + \frac{9}{10}te^{4t} - \frac{3}{100}e^{-6t} - \frac{3}{10}te^{-6t}$$

$$y_{2S} = -\frac{3}{100}e^{4t} + \frac{3}{10}te^{4t} - \frac{1}{100}e^{-6t} + \frac{9}{10}te^{-6t}$$

$$y_1 = y_{1H} + y_{1S}$$

$$y_2 = y_{2H} + y_{2S}$$

RB:

$$\left.\begin{aligned} y_{1(0)} &= 0 = 3C_1 - C_2 + \frac{1}{100} - \frac{3}{100} \\ y_{2(0)} &= 0 = C_1 + 3C_2 - \frac{3}{100} - \frac{1}{100} \end{aligned}\right\} \quad \begin{aligned} C_1 &= \frac{1}{100} \\ C_2 &= \frac{1}{100} \end{aligned}$$

$$\Rightarrow y_1 = \frac{4e^{4t}}{100} + \frac{9te^{4t}}{10} - \frac{4e^{-6t}}{100} - \frac{3te^{-6t}}{10}$$

$$y_2 = \frac{-2e^{4t}}{100} + \frac{3te^{4t}}{10} + \frac{2e^{-6t}}{100} + \frac{9te^{-6t}}{10}$$

DGL-System mit Laplace Transformation

Aufgabe 14.4

$$y_1' - 3y_1 - 3y_2 = e^{4t}$$
$$y_2' - 3y_1 + 5y_2 = e^{-6t}$$

mit $f_1(0) = 0$
$f_2(0) = 0$

$$s F_1 - 3F_1 - 3F_2 = \frac{1}{s-4}$$
$$s F_2 - 3F_1 + 5F_2 = \frac{1}{s+6}$$

$$F_1(s-3) - 3F_2 = \frac{1}{s-4} \quad \Big| \cdot 3$$
$$F_2(s+5) - 3F_1 = \frac{1}{s+6} \quad \Big| \cdot (s-3)$$

$$F_1(s-3) \cdot 3 - 9F_2 = \frac{3}{s-4}$$
$$F_2(s+5)(s-3) - 3F_1(s-3) = \frac{s-3}{s+6}$$

$$F_2(s^2 + 5s - 3s - 15) - 9F_2 = \frac{s-3}{s+6} + \frac{3}{s-4}$$

$$F_2(s^2 + 2s - 24) = \frac{(s-3)(s-4) + 3(s+6)}{(s+6)(s-4)}$$

$$F_2 = \frac{s^2 - 3s - 4s + 12 - 3s + 18}{(s+6)(s-4)(s^2+2s-24)} = \frac{s^2 - 4s + 30}{(s+6)^2(s-4)^2}$$

$$\rightsquigarrow \frac{s^2 - 4s + 30}{(s+6)^2(s-4)^2} = \frac{A}{s+6} + \frac{B}{(s+6)^2} + \frac{C}{s-4} + \frac{D}{(s-4)^2}$$

$$s^2 - 4s + 30 = A(s+6)(s-4)^2 + B(s-4)^2 + C(s+6)^2(s-4) + D(s+6)^2$$

$s = 4$: $16 - 16 + 30 = 30 = D \cdot 100 \Leftrightarrow D = \frac{3}{10}$

$s = -6$: $36 + 24 + 30 = 90 = B \cdot 100 \Leftrightarrow B = \frac{9}{10}$

$$\rightsquigarrow \quad s^2 - 4s + 30 - \frac{9}{10}(s-4)^2 - \frac{3}{10}(s+6)^2 = A(s+6)(s-4)^2 + C(s+6)^2(s-4)$$

$$s^2\left(1 - \frac{9}{10} - \frac{3}{10}\right) + s\left(-4 + \frac{72}{10} - \frac{36}{10}\right) + 30 - \frac{144}{10} - \frac{108}{10} = \dots$$

$$s^2\left(-\frac{2}{10}\right) + s\left(-\frac{4}{10}\right) + \frac{48}{10} = \dots$$

$$-\frac{2}{10}\left(s^2 + 2s - 24\right) = \dots$$

$$-\frac{2}{10}(s+6)(s-4) = \dots \qquad \Big| : (s+6)(s-4)$$

$$-\frac{2}{10} = A(s-4) + C(s+6)$$

$$s = 4: \; -\frac{2}{10} = C \cdot 10 \rightsquigarrow C = -\frac{2}{100}$$

$$s = -6: \; -\frac{2}{10} = A(-10) \rightsquigarrow A = \frac{2}{100}$$

$$\Rightarrow F_2 = \frac{2}{100} \cdot \frac{1}{s+6} + \frac{9}{10}\frac{1}{(s+6)^2} - \frac{2}{100}\frac{1}{s-4} + \frac{3}{10}\frac{1}{(s-4)^2}$$

$$y_2 = f_2 = \frac{2}{100} e^{-6t} + \frac{9}{10} t e^{-6t} - \frac{2}{100} e^{4t} + \frac{3}{10} t e^{4t}$$

$$y_2' - 3y_1 + 5y_2 = e^{-6t}$$

$$\Leftrightarrow y_1 = \left(e^{-6t} - 5y_2 - y_2'\right)/-3$$

$$y_1 = \Big[e^{-6t} - \frac{10}{100} e^{-6t} - \frac{45}{10} t e^{-6t} + \frac{10}{100} e^{4t} - \frac{15}{10} t e^{4t} + \frac{12}{100} e^{-6t} - \frac{9}{10} e^{-6t} + \frac{54}{10} t e^{-6t} + \frac{8}{100} e^{4t} - \frac{3}{10} e^{4t} - \frac{12}{10} t e^{4t}\Big] \cdot \frac{-1}{3}$$

$$y_1 = -\frac{4}{100} e^{-6t} - \frac{3}{10} t e^{-6t} + \frac{4}{100} e^{4t} + \frac{9}{10} e^{4t} \cdot t$$